Detailed Course Contents -MS Mathematics

MATH-801 Algebra

Credit Hours: 3-0

Prerequisite: None

Objectives and Goals: This course will provide a base for many subjects in modern Algebra such as commutative algebra, algebraic geometry, algebraic number theory, computational commutative algebra, multiplicative ideal theory, homological algebra and combinatorial commutative algebra and representation theory.

Core Contents: Groups, group actions and permutation representations, class equation of group, Sylow's theorems, simple groups, direct product and finitely generated abelian groups, rings, ideals, Euclidean domains, principal ideal domains, unique factorization domains.

Detailed Course Contents: Groups, dihedral groups, symmetric groups, matrix groups, the Quaternion group, homomorphism and isomorphism, subgroups generated by a subset of a group, the lattice of subgroups of a group, Fibers of a group homomorphism, quotient groups, group actions, group actions and permutation representations, group acting on themselves by left multiplication, group acting on themselves by conjugation, the class equation, the Sylow's theorems, simple groups, conjugacy in symmetric groups, the fundamental theorem of finitely generated abelian groups, rings, subrings, ideals, polynomial rings, quotient rings, ring homomorphism, properties of ideals, integral domains, prime and irreducible elements, Euclidean domains, principal ideal domains, unique factorization domains, polynomial rings over fields, polynomial rings that are unique factorizations

Course Outcomes: On successful completion of this course, students will

know groups, subgroups, group action, factor groups, number of all possible abelian groups up to isomorphism for a given order, Sylow theorems, application to simplicity of groups, rings, subrings, ideals, polynomial rings, reducible and irreducible polynomials over certain rings, integral domains, Euclidean domains, principal ideal domains, unique factorization domains

Textbook: David S. Dummit, Richard M. Foote, Abstract Algebra, 3rd Ed., John Wiley &Sons.

Reference books

1. N. Herstein, Topics in Algebra, John Wiley and Sons.
2. W. Keith Nicholson, Introduction to Abstract Algebra, (3rd edition), 2007, John Wiley & son

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown

Week	*Section*	*Topics*
1	Sec. 1.2-1.6	Group of symmetries of a geometric object, examples, presentation of agroup, matrix groups, the quaternion group, group homomorphisms
2	Sec. 1.7	Group actions, examples of group actions, permutation representation associated to the given action. faithful and transitive actions.
3	Sec. 2.1,	Subgroups, centralizers, and normalizers. Stabilizers and kernels

	2.2, 2.3	of the group actions. Cyclic groups and cyclic subgroups.
4	Sec. 2.4, 2.5	Subgroups generated by a subgroup of a group, the lattice of the subgroups of a group.
5	Sec. 3.1	Fibers of a group homomorphism and related theorems, quotient group using fibers of a group homomorphism, quotient group by a normal subgroup.
6	Sec. 3.2	Lagrange theorem and its converse, Cauchy's theorem, composition of two subgroups and related results.
7	Sec. 3.3, 4.1	Isomorphism theorems, the correspondence theorem and its applications tofactor group. Group action and permutation representations.
8	Sec. 4.2	Orbit stabilizer theorem, group acting on themselves by left multiplication, Smallest prime index theorem.
9	**Mid Semester Exam**	
10	Sec. 4.3	Group acting on themselves by conjugation, the class equation of a group and applications, conjugacy in $S_{n.}$
11	Sec. 4.5	Proofs of Sylow's theorems using group action, applications of Sylow's theorems to simple groups.
12	Sec. 5.1, 5.2	Direct products, the fundamental theorem for finitely generated abelian groups.
13	Sec. 7.1, 7.2	Rings, matrix ring, group ring, the ring of residue classes modulo *n*, polynomial ring in several variables, integral domains, fields.
14	Sec. 7.3	Ideals, quotient rings, ring homomorphism, isomorphism theorems for rings, the correspondence theorem for rings and applications to quotient rings.
15	Sec. 7.4, 8.1, 8.2	Properties of ideals, characterization of prime and maximal ideals. Norms on integral domains, division algorithms for integral domains, examples,principal ideal domains, examples.
16	Sec. 8.3	Prime and irreducible elements, examples, unique factorization domains, examples.
17		Review

18	**End Semester Exam**